Using Ant Colony Optimization for Sensitivity Analysis in Structural Equation Modeling

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Abstract

Studies using structural equation modeling (SEM) to evaluate theories against observed data rely on multiple sources of evidence to support a proposed model, such as fit indices, variance explained, and comparison of alternative models. Additional evidence can be obtained by evaluating the model results' sensitivity to an omitted confounder. The phantom variable approach to SEM sensitivity analysis requires manual specification of sensitivity parameters. This study improves on the phantom variable approach by employing the ant colony optimization algorithm to automatically search for sensitivity parameters, if any, that would lead to a change in the study's conclusions. The proposed method is implemented in the package SEMsens for the R statistical software, and demonstrated with a sensitivity analysis of a model of the complex relation between working memory and writing.

Using Ant Colony Optimization for Sensitivity Analysis in Structural Equation Modeling

Structural equation modeling (SEM) has been extensively used for testing complex theories based on models that represent predictions from those theories (Hayduk et al., 2007) in a wide range of research areas, such as education, environmental sciences, medical sciences, and the social sciences. There are well-established guidelines for the implementation and reporting of SEM (e.g., Boomsma, 2000; McDonald & Ho, 2002). These guidelines indicate that an important final step of applying SEM is the consideration of alternative models. Preferably, these models would originate from alternative theories (Boomsma, 2000), but they could also come from a specification search (Marcoulides & Falk, 2018). These alternative models only consider relations between variables already in the model, and do not consider whether an omitted confounder could change the conclusions obtained from the model. Omitted confounders, a type of external misspecification (Kaplan, 1990), are variables not in the researcher's model, yet exert a direct effect on variables in the model. Examining sensitivity of the current model to omitted confounders is important because an omitted confounder may lead to incorrect conclusions about theoretical relations involving latent variables as well as observed variables. Providing a sensitivity analysis to omitted confounders may strengthen the validity of statistical conclusions (Shadish et al., 2002), or provide information about model improvements that could be addressed in future research.

A sensitivity analysis is a post-analytic method to determine whether conclusions from a statistical analysis would change if different decisions had been made during the analysis process. The first sensitivity analysis method was proposed by Cornfield et al. (1959) for analyzing the effects of smoking on lung cancer. Since then, there have been numerous sensitivity analytic methods developed for a variety of statistical analyses, such as regression (e.g., Frank, 2000), logistic regression (e.g., Lyles & Lin, 2010), marginal structural models (e.g., Brumback et al., 2004), multiple imputation (e.g., Rezvan et al., 2015), and meta-analysis (e.g., Carpenter, 2010). Within the field of SEM, there have been methods proposed to diagnose sensitivity to omitted model paths between variables already in the model (e.g., Yuan et al., 2008), outliers (e.g., Pek & MacCallum, 2011), and missing data assumptions (e.g., Xu & Blozis, 2010). Only two studies have proposed methods to evaluate sensitivity to omitted variables (e.g., Harring et al., 2017; Tofighi et al., 2019).

In the current study, we propose a new method of sensitivity analysis to evaluate the extent to which inferences drawn from SEM analyses hold firm in the presence of omitted confounders. The method we propose builds on the methods recently proposed by Harring et al. (2017). These methods, one within the frequentist framework and another within the Bayesian framework, involve the inclusion of a phantom variable (Rindskopf, 1984) in the model to represent an omitted variable. These methods' main limitation is the difficulty of a priori specification of the pattern of path coefficients between the phantom variable and the other variables in the model, which can be daunting for large models. In the frequentist approach, the researcher must manually specify candidate values of the phantom variable's path coefficients; while for the Bayesian method, prior distributions must be specified (Harring et al., 2017). This drawback can be overcome with metaheuristic optimization algorithms (Dréo, Pétrowski, Siarry, & Taillard, 2006), such as the ant-colony optimization (ACO) algorithm (Socha & Dorigo, 2008). The main objective of the current paper is to present and demonstrate a new SEM sensitivity analysis method that uses the ACO algorithm for automatic specification of candidate values of the phantom variable's path coefficients.

The current paper has the following structure: 1) Review of SEM sensitivity analysis based on phantom variables; 2) Introduction to the ACO algorithm; 3) Description of how the ACO algorithm was adapted for SEM sensitivity analysis; 4) Description of current features implemented in the *SEMsens* package of the R statistical software, which performs SEM sensitivity analysis with the ACO algorithm; 5) Illustrative example of SEM sensitivity analysis with the *SEMsens* package; 6) Discussion of the value of SEM sensitivity analysis for the current example and for SEM researchers in general; 7), Conclusions with limitations of the current method and future avenues for research.

Sensitivity Analysis in Structural Equation Modeling

The Phantom Variable Method

Sensitivity analysis for structural equation modeling against a missing confounder can potentially be tracked through a phantom variable approach (Harring et al., 2017). This approach first specifies a phantom variable that confounds the relations among variables already in the model. A phantom variable is a latent variable without manifest indicators but with mean, variance, covariances, and paths to variables in the model set to specific values (Rindskopf, 1984). The path coefficients from the phantom variable to variables in the analytic model can be viewed as the sensitivity analysis parameters, quantifying the hypothetical relations between a potential confounder and variables in the model that could change the statistical conclusions of the model. Varying these parameters can give researchers the ability to investigate the sensitivity of the results to a missing confounder.

INSERT FIGURE 1 ABOUT HERE

Using a simple structural model of mediation, we illustrate the difference in implied covariance matrices for models with and without a phantom variable. Using path tracing rules, the analytical model for simple mediation (see Figure 1a) has the following implied covariance matrix for variables X, M and Y:

$$\Sigma = \begin{bmatrix} \phi_{11} & & \\ \beta_1\phi_{11} & \beta_1^2\phi_{11} + \psi_{22} & \\ \beta_2\beta_1\phi_{11} & \beta_2(\beta_1^2\phi_{11} + \psi_{22}) & \beta_2^2(\beta_1^2\phi_{11} + \psi_{22}) + \psi_{33} \end{bmatrix}$$

Including the phantom variable (see Figure 1b) changes the implied covariance for variables X, M and Y to:

$$\Sigma =$$

$$\begin{bmatrix} \gamma_1^2 + \psi_{11} & & \\ \beta_1(\gamma_1^2 + \psi_{11}) + \gamma_1\gamma_2 & \beta_1^2(\gamma_1^2 + \psi_{11}) + \gamma_2^2 + \psi_{22} & \\ \beta_2\beta_1(\gamma_1^2 + \psi_{11}) + \gamma_1\gamma_3 & \beta_2[\beta_1^2(\gamma_1^2 + \psi_{11}) + \gamma_2^2 + \psi_{22}] + \gamma_2\gamma_3 & \begin{matrix}\beta_2^2[\beta_1^2(\gamma_1^2 + \psi_{11}) + \\ \gamma_2^2 + \psi_{22}] + \gamma_3^2 + \psi_{33}\end{matrix} \end{bmatrix}$$

Comparing the implied covariance matrices for models without and with an omitted confounder (i.e., the phantom variable), each of the corresponding elements in the two implied covariance matrices are unequal when sensitivity parameters are not zero (i.e., $\gamma_i \neq 0$ for *i* = 1, 2, and 3). In general, each element in the model-implied matrix for a sensitivity analysis includes at least one sensitivity parameter. For example, for the covariance between variables X and M, the elements in the covariance matrix for a sensitivity model includes γ_1 and γ_2, or the respective sensitivity parameters from the phantom variable to variables X and M. In a same vein, the variance components in the implied covariance matrix for a sensitivity model also include sensitivity parameters. The variance decomposition of M for example, includes the additional terms γ_1 and γ_2. The implication is that, as the values of sensitivity parameters (γs) move away from zero, the estimated paths coefficients (i.e., βs) and associated p-values will depart from their respective values in the initial analytical model. Because most SEM fit indices are based on the fitted model, these changes cause the fit indices for the whole model to change as well.

In a sensitivity analysis, those sensitivity parameters which may change the statistical conclusions of the study are of particular importance. For example, a researcher may hypothesize a specific positive direct effect of an exogenous variable in the model, as well as an indirect effect, and find in the model results that the study's hypotheses hold. However, through a sensitivity analysis, the researcher may find that small sensitivity parameters corresponding to an unmeasured confounder result in the direct effect changing from significant to not significant, while the significance test of the indirect effect does not change for a wide range of values of the sensitivity parameters. In this case, the authors could conclude that future research should re-examine whether the relation between the exogenous and endogenous variable is partially or fully mediated while controlling for additional potential confounders.

In general, for an SEM sensitivity analysis, researchers can draw the conclusion that the model is sensitive to a potential missing confounder if small sensitivity parameters invalidate the results of an analytic model. On the other hand, if only large sensitivity parameters change the conclusions, this indicates that the model is robust to a potential missing confounder. A combination is also possible, where some paths are insensitive, and other paths are sensitive to, a potential missing confounder. Thus, in many situations, researchers should assess the sensitivity to a missing confounder for each path in an SEM by reporting the sensitivity parameters that make substantial changes to the estimates (e.g., significance level).

Harring et al. (2017) proposed a frequentist and a Bayesian approach for SEM sensitivity analysis with phantom variables. They then demonstrated both approaches in a model proposed by Sava (2002) where a potential unobserved confounder (i.e., student negative behavior) had direct effects on the endogenous variables and correlated with the exogenous variables in the model. For the frequentist approach, Harring et al. (2017) chose specific values for the sensitivity parameters, while for the Bayesian approach they chose means and variances of informative prior distributions. For the frequentist approach, they recommended trying many different combinations of sensitivity parameters. This approach was applied by Leite et al. (2019), who defined three standardized values (i.e., 0.1, 0.25, and 0.5) of three different sensitivity parameters to create nine models with a phantom variable. After fitting these nine models, Leite et al. (2019) plotted the *p*-values of the parameter estimates for the combinations of sensitivity parameters.

Manually specifying sensitivity parameters, as done by Harring et al. (2017) and Leite et al. (2019), is complicated for even fairly simple structural equation models. The number of possible combinations of sensitivity parameters for a phantom variable increases exponentially as variables enter the model. Furthermore, each sensitivity parameter can take on an infinite number of possible values. Even if we discretize this continuous domain (e.g., from -0.9 to 0.9 with an incremental value of 0.1), there would still be 19^n possible solutions (*n* being the number of variables connected to the unmeasured confounder). For an SEM model with five variables, this results in 2.5 million possible solutions. Comparing all possible solutions quickly becomes infeasible as the number of variables in an SEM model gets larger.

Other Methods for Sensitivity Analysis in Structural Equation Modeling

To our knowledge, there is another sensitivity analysis approach investigating the robustness of SEM models against misspecification. Kolenikov (2011) proposed a framework to link moment residuals with biases of parameter estimates and the overall noncentrality of a model. This method can assess the bias introduced to parameter estimates by internal misspecification. That is, all causally relevant variables are included in the analytical models to be compared, but some path configurations may be incorrect. This method allows researchers to compare different specified models and assess the bias in path coefficients introduced by a misspecifed model. To implement the procedure, researchers must manually specify two different models that include the same variables.

In the field of causal graphs (Pearl, 1998), which includes SEM as a special case, the Tetrad software has been developed as a modular program which can generate an equivalence class of causal graphs with data through different search algorithms (Landsheer, 2010; Ramsey & Malinsky, 2017). When using Tetrad, the procedure begins with the researcher feeding data into a search algorithm, then choosing assumptions to produce an equivalence class of graphs as outputs. The researcher then chooses a corresponding parametric model that makes sense for the data and causal graphs to assign values to parameters (Haughton et al., 2006; Landsheer, 2010; Ramsey & Malinsky, 2017). Tetrad uses the graph with bidirected edges to indicate that two variables have unmeasured correlated error terms due to omitted confounders. When using the selected model to assign values to parameters, the values are generated by Tetrad randomly rather than generated by search algorithms (Ramsey & Malinsky, 2017). While Tetrad could tell researchers if there is a potential for an omitted confounder in different parts of a model, the SEM sensitivity analysis method based on the ACO algorithm that we developed is able to quantify how strongly an omitted confounder must be related to current variables in the model to change the conclusions obtained with the model. In the next section we overview the ACO algorithm before providing details on its application to sensitivity analysis.

The Ant Colony Optimization Algorithm

The ACO algorithm (Colorni et al., 1992) was inspired by the behavior of ant colonies, which using a simple set of rules, are able to converge to the shortest path from the colony to the food source. Ants accomplish this by leaving a trail of pheromone (i.e., scent) behind them as they walk, which guides the behavior of subsequent ants as they choose which direction to walk. This occurs because ants take more time to travel down a longer path and back again than a shorter path, and the more time spent on traveling on a longer path leads to more pheromone evaporation. Consequently, pheromone accumulates faster in shorter paths than in longer paths. Thus, longer paths become less attractive to other ants than shorter paths (Dorigo & Stültze, 2004).

The ant colony optimization algorithm utilizes artificial ants to move through a parameter space that represents all possible solutions. This optimization method has been used in a variety of applications (Dorigo & Stültze, 2004). For example, the traveling salesman problem is a classical combinatorial problem often used to test new optimization methods. The travelling salesman problem asks the following question: Given n cities and the distances between all pairs of cities, what is the shortest possible route that travels each city and returns to the origin city? The total number of solutions increases exponentially as the number of cities gets larger, which makes searching for all combinations of the solution space computationally infeasible. Within the field of structural equation modeling, the ACO algorithm has been used for model specification search (Marcoulides & Leite, 2012), and for selecting items for short forms of scales that optimize certain results of confirmatory factor analysis, such as model fit and magnitude of target parameter estimates (Leite et al., 2008; Raborn & Leite, 2018; Raborn et al., 2020).

The ACO algorithm consists of sampling an initial set of solutions and evaluating them. In this initial stage, the probabilities of components across all solutions are the same. Based on the quality of each solution, the probabilities of the components that lead to better solutions are increased by a certain amount, which is referred to as the deposit of pheromone. This is repeated until the specified termination criterion is achieved. Several strategies have been implemented in ACO algorithms to prevent local optima. One such approach is manipulating pheromone evaporation (i.e., the pheromone already accumulated from previous runs slowly decreases), which has been shown to be quite effective in avoiding getting trapped in local solutions (Leite et al., 2008).

The ACO algorithm was originally proposed to solve combinatorial problems for which the components are discrete in nature (e.g., Colorni et al, 1992; Dorigo & Stützle, 2004). A combinatorial problem is usually decomposed into a finite set of components, and the algorithm tries to find the optimal combination. For example, in the traveling salesman problem with n cities, there are $(n-1)!$ unique paths that traverse every city. Some optimization problems have components that are continuous in nature. For example, an optimization problem may require choosing values from continuous variables. In this case, the total number of possible solutions is infinite. Admittedly, some continuous optimization problems may be discretized and solved with the original (discrete) ACO algorithm. Depending on how many significant figures are necessary, such dimension reduction may cause the optimal solution to be left out of the discrete domain (i.e., the optimal solution in the continuous domain is not included in the discrete domains). Furthermore, it is not always convenient to convert a continuous optimization problem into a discrete one. For these reasons, the ACO algorithm has been generalized to continuous domains (Socha & Dorigo, 2008).

In the ACO algorithm, it works through a discrete probability distribution for a finite set of available components (or solutions) that changes over iterations. Better solutions will have a larger probability of being sampled, resulting from the accumulated pheromone. In the probability update process, the optimal solutions are eventually found. The ACO algorithm with continuous domains works the same way but with a continuous probability density function over the possible range (see technical details in Socha & Dorigo, 2008). Termination conditions can be customized by the user, but as with any metaheuristic algorithm (Dréo et al., 2006), there is no guarantee that the final solution is the overall best solution.

Sensitivity Analysis in SEM Using the ACO Algorithm

We applied the version of the ACO algorithm for continuous domains developed by Socha and Dorigo (2008) to search for sensitivity parameter values, if any, that change the conclusions of an SEM model. The conclusions of an SEM model that can be probed by a sensitivity analysis are related to statistical significance tests, size of parameter estimates for certain paths, and model fit (Harring et al., 2017). The ACO algorithm can be used for SEM sensitivity analysis through the following steps, which are summarized in Figure 2: (1) From the complete results of a structural equation model, the researcher decides on target paths, which are paths that should be assessed for sensitivity analysis against a potential missing confounder. This could be all estimated path coefficients or a smaller subset. (2) The researcher sets up the sensitivity analysis model, which is the original model plus paths from a phantom variable to variables in the model. (3) The researcher selects an objective function, which defines the target for optimization; Objective functions can be defined based on model coefficient estimates, *p*-

values, and fit indices. Objective functions encode the type of changes being examined that could invalidate the conclusion of a structural equation model. For example, an objective function based on p-values would examine changes in the p-values of parameter estimates in the model from significant to non-significant. 4) The ACO algorithm runs until it terminates. Termination is pre-specified by the researcher and occurs when (a) a maximum number of evaluations has been reached; (b) an optimal value of the objective function has been achieved; or when (c) no changes in p-values have been observed after a certain number of iterations. At termination of the algorithm, the researcher can summarize the results in tables and plots and make interpretations as final steps to the SEM sensitivity analysis.

INSERT FIGURE 2 ABOUT HERE

Next, we explain some of the technical details about the optimization process of the ACO algorithm for SEM sensitivity analysis. The search of the sensitivity parameters that change the conclusions of a structural equation model can be converted into an optimization problem using ACO for continuous domains (Socha & Dorigo, 2008). An objective function for optimization in the sensitivity analysis can be defined as

$$Q = (S_n, f),$$

where S_n is an initial search space defined over a set of n continuous sensitivity parameters with a set of n constraints among them (e.g., -1 to 1 as the initial domain for each sensitivity parameter), f is the objective function. An example objective function to be maximized can be

$f = \frac{1}{\boldsymbol{p_S^*} - \mathbf{0.05}}$ with $\boldsymbol{p_S^*}$ as a vector of p-values for paths in the sensitivity analysis model, the initial search space S_n can be defined as (-1, 1) for each sensitivity parameter.

Once we define the objective function and an initial search space, the next step is to randomly select k (e.g., 50; $k \geq n$) combinations of sensitivity parameters from the initial search space, where k is the length of the solution archive. The value of the objective function is then calculated for each of the k combinations of sensitivity parameters. The algorithm then forms a probability density function across each sensitivity parameter according to the rank of values in the objective function. A higher-ranking value of the objective function results in a sensitivity parameter having a higher probability of being sampled in the next iteration.

The next step is to let the artificial ants run through the search space. Each ant will sample a combination of sensitivity parameters according to the probability density function. Then, the values of the objective function are calculated from sensitivity analysis models (i.e., models with sensitivity analysis parameters). The next step is the pheromone update in the solution archive by (1) ranking the obtained values of the objective function in this iteration and those that are already in the solution archive; (2) keeping the k best values of the objective function; and (3) updating the probability density function according to the updated rank of solutions and sets of sensitivity parameters. The next iteration is then performed with this updated probability density function. The algorithm will stop when one of following termination criteria is reached: (a) the maximum number of iterations; (b) the maximum value of objective function is reached if it exists; (c) the best solution values do not change over certain number of iterations (e.g., 100).

Illustrative Example

In this section, we demonstrate the SEM sensitivity analysis method using the ACO algorithm to automatically search for specifications of path coefficients of the phantom variable and values of the sensitivity parameters. This method is currently available in the *SEMsens* package of the R statistical software (R Development Core Team, 2020). We performed a sensitivity analysis for a structural equation model (see Figure 3) published from an IES (Institute of Education Sciences)-funded study that examined the relation between working memory and writing (Kim & Schatschneider, 2017). We chose this study because it evaluated a complex model with several indirect effects involving multiple mediators between working memory and writing, which would not be easily amenable to SEM sensitivity analysis by manual specification of sensitivity parameters. The model chosen represents the type of model found in applications of SEM. The authors were particularly interested in finding whether there are direct effects from working memory, vocabulary, grammar, inference, and theory of mind (ToM) to writing. They compared models with and without direct effects using chi-square difference tests and concluded that the full mediation model fit the data as well as models that also included direct effects.

The SEM sensitivity analysis was specified with a potential missing confounder with direct effects on working memory and writing, as well as direct effects on all mediators of the relation between working memory and writing. Thus, the analysis included 9 sensitivity parameters, as shown by the dashed arrows on Figure 3.

INSERT FIGURE 3 ABOUT HERE

For this example, we used the following objective function defined on p-values:

$$y = \frac{1}{\frac{1}{J}\sum_{j=1}^{J} |p_*^j - p_{sig}|}, \quad (1)$$

where p_*^j is the estimated p-value of path j in a model with sensitivity parameters, and p_{sig} is the significance level users specify (0.05 was used in this paper). The denominator is the average absolute difference between the estimated p-values and the significance level across all paths ($J = 18$ in our case, see Table 1). The purpose was to identify the sensitivity parameters that led to a change in significance across multiple paths in the authors' final model.

The objective function y now is converted to a maximization problem that can be solved through the *sa.aco* function of the *SEMsens* package. There are six pre-defined objective functions implemented in the package, which can perform a sensitivity analysis using predefined optimization formulas based on p-values, size of path coefficients, or Root Mean Squared Error of Approximation (RMSEA), using any part of the results of a SEM model fit with the *lavaan* package (Rosseel, 2012). Also, the *sa.aco* function allows users to customize any objective function to include any number of paths. The code to produce the results is available in the online supplemental materials.

Results

The major advantage of metaheuristic algorithms over brute force search is that they do not require evaluating all possibilities to find a solution. For this example, it took less than 4 minutes to perform the sensitivity analysis, which took 1,006 evaluations on one core of a multicore laptop. The convergence rate of models fit during the search was 93.2%. A low convergence rate may produce blind spots in the sensitivity analysis, as we will discuss later. Table 1 summarizes the results, ordered so that paths with a *p*-value that changed from significant to non-significant or vice-versa are shown first, and then ranked with respect to the mean percentage change in the parameter estimate relative to the parameter estimate in the original model. Following Kolenikov's (2011) recommendation that parameters with less than 10% change can be considered insensitive to misspecification, we marked in bold the parameters that had a mean percentage change of 10% or greater. In Table 1, the Y~X notation indicates a path from X to Y. The results show that several path coefficients (e.g., Discourse~Grammar, Discourse~ToM, ToM~Working_Memory) are sensitive to a potential confounder as reflected by the change in *p*-value as well as a large mean percentage change. One implication of this result is that the relation between grammar and discourse could be fully mediated by the Inference variable, rather than being partially direct and partially mediated through ToM. On the other hand, several other path coefficients (e.g., Discourse~Working_memory, Sentence_copying~Working_memory, Discourse~Vocabulary) are insensitive to an omitted confounder as reflected by no change in *p*-values, means of path coefficients identical to their original estimate, and relatively narrow ranges (i.e., Minimum to Maximum) around means. There were also paths that did not change the *p*-value during the search, but showed a substantial mean percentage change in the parameter estimate (e.g., ToM~Grammar, Inference~Grammar), indicating that even though the parameter estimate may be sensitive to misspecification, the conclusion with respect to the significance of the parameter estimate is not sensitive.

INSERT TABLE 1 ABOUT HERE

The *sens.tables* function of the *SEMsens* package also provides the sensitivity parameters that led to the change in significance[1], as shown in Table 2. Although there are no benchmarks for SEM sensitivity parameter size, to facilitate comprehension we put in bold the sensitivity parameters with absolute value at or above 0.2, and underlined those at or below 0.10, while those between 0.1 and 0.2 are in regular font. Inspection of Table 2 shows that the change in *p*-values is associated with confounders with strong positive relationships with ToM, Spelling and Writing, and large negative relationships with Grammar. In contrast, the confounder had weak positive relations with Working Memory and Spelling and a weak negative relation with Vocabulary.

INSERT TABLE 2 ABOUT HERE

Table 3 provides the summary of sensitivity parameters, including the mean, minimum, maximum, and range of values for each of the sensitivity parameters across the search. The values are ranked by the absolute value of the mean. The results show that not all paths contribute equally to the optimization of the objective function based on *p*-values. It shows that the paths of the confounder on ToM, Writing, and Spelling contributed more to the maximization of the objective function as reflected by their relatively larger deviance from zero. This

[1] The *sens.tables* function also provides two tables which report the sensitivity parameters that lead to the minimum and maximum path coefficients

information matches the conclusion from Table 2 that confounders with strong associations with ToM, Writing, and Spelling are more likely to result in a change in *p*-values. Therefore, we can conclude that the results of the study are least sensitive to confounders of ToM, Writing, and Spelling, because it would take strong confounders of these values to change the *p*-values.

INSERT TABLE 3 ABOUT HERE

Discussion

The contribution of an SEM study to the development and improvement of theory is conditional upon the validity evidence that is collected about the model. Even with various fit indices assisting researchers in model selection (Marsh et al., 2005; Yuan et al., 2016; Marcoulides & Yuan, 2017), it is still possible that an SEM study may omit a potential confounder due to (a) the incompleteness of previous theories guiding a current study, or (b) the data set used by an SEM study simply has a limited number of variables. Thus, it is important to assess how firmly the conclusion of an SEM study can hold against an omitted confounder. Such assessment can provide the evidence of the validity of an SEM study and provide applied researchers with informative clues to identify a potential missing confounder and further advance the development of the underlying theory.

This article presents the implementation of a sensitivity analysis approach to structural equation modeling against a potential missing confounder. Our approach combines the recent development of an SEM sensitivity analysis method based on phantom variables and a metaheuristic optimization algorithm. This innovative integration allows us to explore the robustness of an SEM model against a potential missing confounder, providing researchers with an additional tool to assess the validity of an SEM study. In addition, we have implemented the proposed sensitivity analysis approach in the R package *SEMsens*, which we believe will make this approach more accessible to applied researchers.

The example presented demonstrates that SEM sensitivity analysis can produce two types of results: (1) The sensitivity analysis shows that it would take a confounder in certain positions of the SEM system of equations with very strong coefficients for the conclusions of the model to change. For this result, it is recommended that researchers add a description in the paper's discussion section about the possibility of such a confounder based on their knowledge and expertise. This result is evidence of statistical conclusion validity (Shadish et al., 2002), and in the case of measurement studies, evidence of validity with respect to relations with other variables and internal structure (American Educational Research Association et al., 2014). (2) The sensitivity analysis shows that confounders with weak to moderate relations in certain positions of the system would change model conclusions. For this result, researchers should use their knowledge and expertise to discuss candidates for such confounders that would be included in the model in additional analyses or future studies. This result could also prompt the researchers to consider broader theoretical revisions of the model which should include additional variables.

Conclusion

The current approach for SEM sensitivity analysis is to search a combination of several sensitivity parameters that may change the conclusions of a study while optimizing on objective function of p-values, estimated path coefficients, or model fit indices. While in this study we only used one objective function based on *p*-values, researchers could choose a combination of criteria to optimize. For example, in an application of the ACO algorithm to SEM for selecting indicators for scale short forms, Leite et al. (2008) optimized an objective function of three fit indices and the path coefficient of a single covariate.

In the SEM sensitivity analysis method presented, there is no assumption about what the true model is. Rather, we view each of the models with sensitivity parameters as an alternative model. By assessing the impact of the configuration of sensitivity parameters, this approach provides how robust an SEM model is against a potentially omitted confounder. Because of the exploratory nature of the search, this approach does not provide an estimate of bias due to misspecification, which can be obtained with the Kolenikov (2011) approach for diagnosing misspecification.

The ACO algorithm, like any metaheuristic algorithm, does not guarantee that an optimum solution is found (Dréo et al., 2006). Also, because the algorithm needs starting sets of sensitivity parameters, it is possible that we may have different results across runs with different starting sets when a limited space is explored in each run. Additionally, the algorithm may potentially push some of the sensitivity parameters toward extreme values, which may cause non-convergence or an improper solution for some structural equation models. Therefore, the space of sensitivity parameters that result in non-convergence or improper solutions may not be adequately explored by the ACO algorithm, limiting the results of the sensitivity analysis. For these reasons, we suggest researchers to run the algorithm multiple times to check whether similar results can be reached, and to save the random seeds of each run so that the sensitivity analysis results can be reproduced. However, Tofighi et al. (2019) indicated that in some cases, models with phantom variables will produce improper solutions regardless of the values of the sensitivity parameters. To address this situation, they proposed a re-parameterization of the model with phantom variable into an equivalent model where the phantom variable and associated paths are replaced by residual correlations. After the sensitivity analysis, the residual correlations can then be converted back to path coefficients of the phantom variable. It is feasible to incorporate Tofighi et al.'s proposed model re-parameterization into the SEM sensitivity analysis with the ACO algorithm. In future developments of the R package, models resulting in improper solutions may be re-parameterized and re-fit.

The proposed SEM sensitivity analysis is implemented within a frequentist framework, but extensions of Harring et al. (2017) Bayesian SEM sensitivity analysis should also be investigated. Although the SEM sensitivity analysis with the ACO algorithm presented here could be used for any type of omitted variables, it is currently only implemented for a single latent continuous omitted confounder. Future research could explore SEM sensitivity analysis to latent discrete confounders, where the assumption by the researcher is that the sample originates from a single distribution is evaluated by simulating finite mixtures (McLachlan & Peel, 2000). This method could then be used both as SEM sensitivity analysis method and a sensitivity analysis for class enumeration in mixture SEM.

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Tables

Table 1

Summary of Sensitivity Analysis

Paths	**Est**	***p***	***p****	**Mean**	**Min**	**Max**	**Mean % Change**
Discourse~Grammar	0.10	0.25	0.05	0.18	0.05	0.24	**80.00%**
Discourse~ToM	0.26	0.00	0.05	0.16	0.07	0.31	**38.46%**
ToM~Working_memory	0.20	0.00	0.05	0.14	0.08	0.25	**30.00%**
ToM~Vocabulary	0.26	0.00	0.05	0.19	0.14	0.31	**26.92%**
Writing~Sentence_copying	0.17	0.04	0.05	0.15	0.05	0.27	**11.76%**
Inference~Working_memory	0.12	0.03	0.05	0.11	0.08	0.15	8.33%
Discourse~Inference	0.19	0.04	0.05	0.20	0.15	0.25	5.26%
Writing~Discourse	0.45	0.00	0.06	0.35	0.17	0.60	**22.22%**
ToM~Grammar	0.28	0.00	--	0.39	0.20	0.45	**39.29%**
Inference~Grammar	0.29	0.00	--	0.32	0.24	0.35	**10.34%**
Spelling~Working_memory	0.39	0.00	--	0.36	0.32	0.41	7.69%
Writing~Spelling	0.38	0.00	--	0.36	0.19	0.66	5.26%
Inference~Vocabulary	0.43	0.00	--	0.41	0.41	0.46	4.65%
Vocabulary~Working_memory	0.40	0.00	--	0.40	0.39	0.41	0.00%
Grammar~Working_memory	0.40	0.00	--	0.40	0.36	0.42	0.00%
Discourse~Vocabulary	0.26	0.00	--	0.26	0.23	0.33	0.00%
Sentence_copying~Working_memory	0.24	0.00	--	0.24	0.23	0.26	0.00%

Discourse~Working_memory	0.19	0.01	--	0.19	0.15	0.23	0.00%

Note. *Est* is the standardized path coefficient in the original analytic model, p is the p-value in the original analytic model, p^* is the closest p-value to significance level in a sensitivity analysis model, *Mean* is the mean value of standardized path coefficient across the search, *Min* is the minimum value of standardized coefficient across the search, and *Max* is the maximum value of standardized coefficient across the search. *Mean % change* is the absolute value of the difference between *Est* and *Mean* divided by *Est*. Values of *Mean % change* greater than 10% are in bold.

Table 2

Sensitivity Parameters That Led to a Change in Significance

Paths	Sensitivity Parameters								
	SP1	**SP2**	**SP3**	**SP4**	**SP5**	**SP6**	**SP7**	**SP8**	**SP9**
Discourse~Grammar	<u>0.07</u>	**-0.20**	<u>-0.07</u>	**0.55**	0.15	**0.28**	0.10	0.16	**0.44**
Inference~Working_memory	<u>0.03</u>	**-0.20**	<u>-0.07</u>	**0.54**	0.13	**0.31**	<u>0.08</u>	0.16	**0.49**
ToM~Vocabulary	<u>0.01</u>	**-0.22**	<u>-0.04</u>	**0.62**	0.13	**0.32**	0.11	0.17	**0.32**
ToM~Working_memory	<u>0.03</u>	**-0.20**	<u>-0.06</u>	**0.57**	0.15	**0.30**	<u>0.10</u>	0.18	**0.35**
Discourse~ToM	<u>0.03</u>	**-0.20**	<u>-0.06</u>	**0.58**	0.14	**0.30**	<u>0.10</u>	0.18	**0.34**
Writing~Sentence_copying	<u>0.03</u>	**-0.20**	<u>-0.06</u>	**0.57**	0.15	**0.29**	<u>0.10</u>	0.18	**0.35**
Writing~Discourse	<u>0.07</u>	0.12	<u>-0.02</u>	**0.28**	0.16	**-0.19**	<u>0.05</u>	<u>0.09</u>	**0.60**
Discourse~Inference	<u>-0.05</u>	-0.11	<u>-0.02</u>	**0.25**	0.10	**0.11**	<u>0.06</u>	0.13	**0.30**

Note. SP1 = Working_memory~phantom, SP2 = Grammar~phantom, SP3 = Vocabulary~phantom, SP4 = ToM~phantom, SP5 = Inference~phantom, SP6 = Spelling~phantom, SP7 = Sentence_copying~phantom, SP8 = Discourse~phantom, SP9 = Writing~phantom.

Table 3

Summary of Sensitivity Parameters

Sensitivity Parameters	Mean	Min	Max	Range
ToM~phantom	0.55	-0.23	0.79	1.02
Writing~phantom	0.3	-0.53	0.62	1.15
Spelling~phantom	0.28	-0.2	0.45	0.65
Grammar~phantom	-0.19	-0.26	0.18	0.44
Discourse~phantom	0.16	-0.13	0.31	0.44
Inference~phantom	0.13	-0.16	0.36	0.52
Sentence_copying~phantom	0.1	-0.06	0.26	0.32
Vocabulary~phantom	-0.05	-0.16	0.06	0.22
Working_memory~phantom	0.02	-0.21	0.18	0.39

Figures

Figure 1

Conceptual Schematic of Structural Equation Model

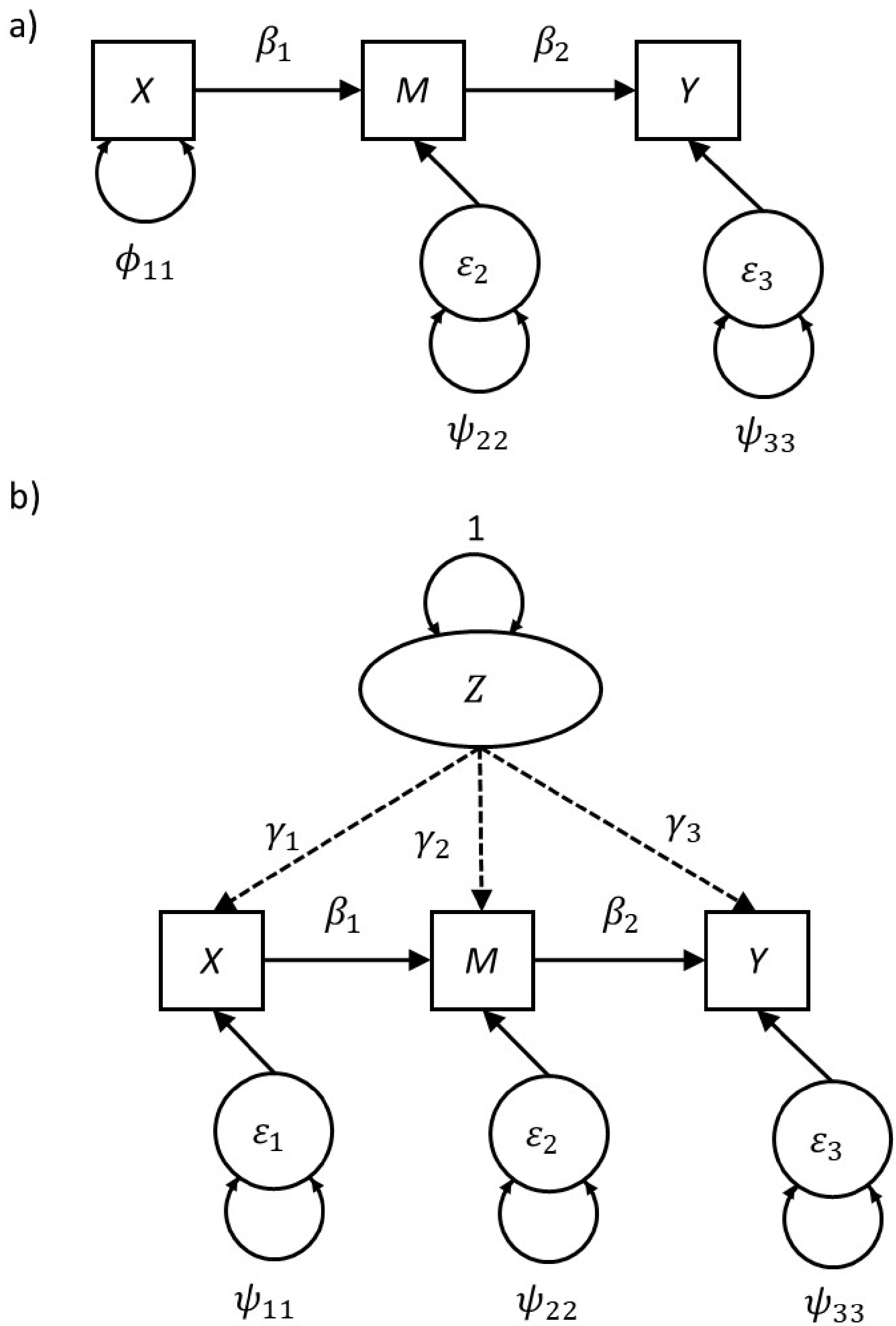

Figure 2

Flowchart of the SEM sensitivity analysis process with the ACO Algorithm

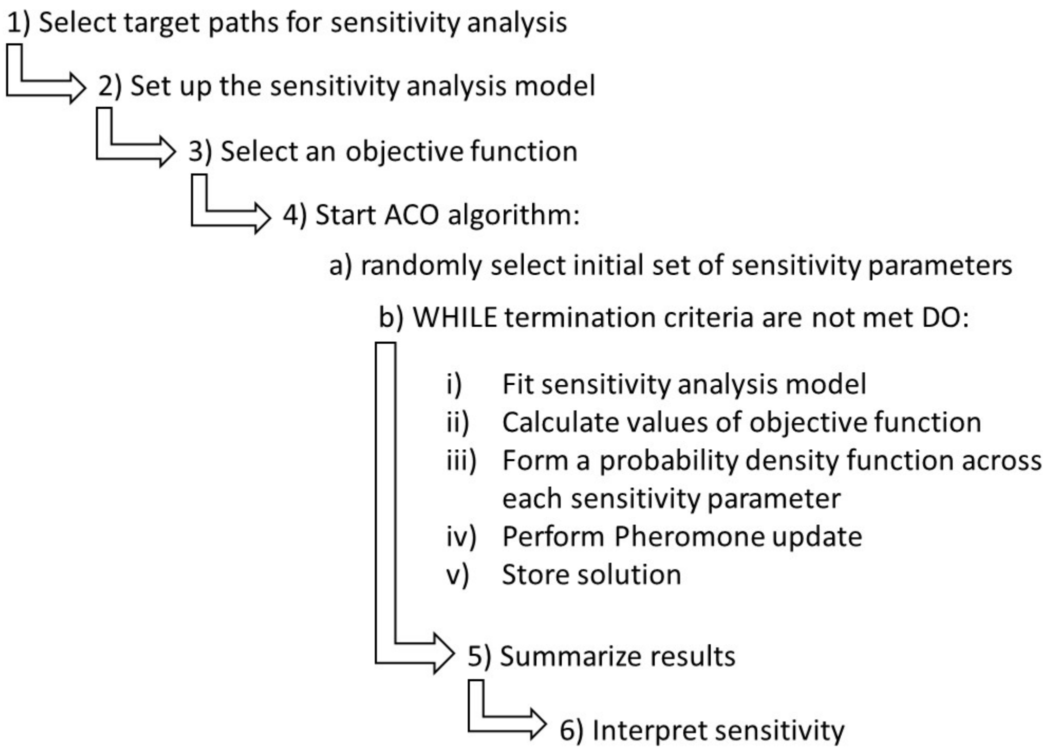

Figure 3

An Example of SEM Sensitivity Analysis Model Based on Kim & Schatschneider (2017)